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Optimal suction of the boundary layer taking account of initial turbulence and surface roughness

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In the paper, on the basis of the three-moment equations, simple formulae for properties of a laminar boundary layer with suction are obtained. An equation relating the transition point critical Reynolds number to initial turbulence and surface roughness is presented. An approximate method for prediction of optimal suction of a boundary layer, including the main effects controlling transition of a laminar boundary layer into a turbulent one, is developed.

Nomenclature

x	Co-ordinate along airfoil.
x_t	Co-ordinate of the point of transition with suction of boundary layer.
x_{t_0}	Co-ordinate of the point of transition without suction of boundary layer.
x_0	Co-ordinate of the point of commencement of boundary-layer suction.
y	Co-ordinate along the normal to airfoil surface.
U_0	Incoming flow velocity.
$U(x)$	Distribution of longitudinal velocity at the outer edge of boundary layer.
$u(y)$	Distribution of longitudinal velocity across boundary layer.
$v_0(x)$	Distribution of normal velocity along airfoil surface (local suction velocity).
$\delta(x)$	Boundary-layer thickness.
$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$	Momentum thickness.
$\delta^{*} = \int_0^\delta \left(1-\frac{u}{U}\right)dy$	Displacement thickness.
δ_t^{**}	Momentum thickness at the point of transition.
δ_d^{**}	Momentum thickness at the roughness element site.
δ_0^{**}	Momentum thickness at the point of commencement of boundary-layer suction.
λ_1	'Smallest vortex' dimension due to external flow turbulence.
λ_2	'Smallest vortex' dimension due to vortices behind roughness element.

d Roughness element height.

L Characteristic dimension (length) of airfoil.

u_0' Root mean square velocity fluctuation of incoming flow.

u_1' Root mean square velocity fluctuation at the outer edge of boundary layer due to external flow turbulence.

u_2' Root mean square velocity fluctuation in the external boundary layer due to vortices behind roughness element.

$p(x)$ Pressure distribution at the outer edge of boundary layer.

$\left[\frac{\partial \tilde{p}}{\partial x}\right]$ Root mean square fluctuation of longitudinal pressure gradient at the outer edge of boundary layer.

$\left[\frac{\partial \tilde{p}}{\partial x}\right]_1$ and $\left[\frac{\partial \tilde{p}}{\partial x}\right]_2$ Components caused by external flow turbulence and vortices behind roughness element, respectively.

τ_w Local shear stress on airfoil surface.

ν Kinematic viscosity.

ρ Liquid density.

$Re = \frac{UL}{\nu}$ The Reynolds number.

$Rx_0 = \frac{Ux_0}{\nu}, R_0^{**} = \frac{U\delta_0^{**}}{\nu}, \quad R^{**} = \frac{U\delta^{**}}{\nu}$ and $R_x = \frac{Ux}{\nu}$ The local Reynolds numbers.

R_H^{**} and R_{x_0} Lower critical Reynolds numbers.

R_t^{**} Critical Reynolds numbers at the point of transition.

$t^{**} = \frac{v_0 \delta^{**}}{\nu}$ Parameter of boundary-layer suction.

$f = \frac{dU}{dx}\frac{\delta^{**2}}{\nu}$ Shape factor of boundary layer.

f_s Shape factor of boundary layer at the point of separation.

$H = \frac{\delta^*}{\delta^{**}}$ Boundary-layer parameter.

$\zeta = \frac{\tau_w \delta^{**}}{\nu \rho U}$ Local coefficient of friction.

ζ_f Coefficient of laminar friction.

$\epsilon = \frac{u_0'}{U_0}$ Initial turbulence.

Constant quantities: $a = 0{\cdot}44$; $b = 5{\cdot}48$; $A_0 = 0{\cdot}22$; $B_0 = 0{\cdot}21$; $B = 1{\cdot}12$; $A_1 = 31{\cdot}3$; $B_1 = 10$; $c = 9{\cdot}55$; $C_1 = 1{\cdot}88$; $C_2 = 1{\cdot}18$; $D_1 = 0{\cdot}25 \times 10^{-3}$; $d = 0{\cdot}56$; $H_0 = 2{\cdot}59$; $H_4 = 4$; $f_{s_0} = 0{\cdot}089$; $\zeta_0 = 0{\cdot}22$; $k = 0{\cdot}194$; $m = 7{\cdot}55$.

Theory

A numerical solution of the problem on optimal suction of fluid from a boundary layer was originally obtained by Pretsch (1943). An analytical solution of the same problem was reported by the author (Kozlov 1963). Also Wieghardt (1954), Wortmann (1958) and Kozlov (1964) worked on the solution of a similar problem for a boundary layer with longitudinal external pressure gradient. The last three works are based on momentum and energy equations and on 'three moment' equations, respectively. In these works the optimal liquid suction from the boundary layer is assumed to be the distribution of the normal velocity over the airfoil surface when, in every cross-section of the boundary layer, the local Reynolds number is equal to its lower critical value calculated by the method of small disturbances developed in the theory of hydrodynamic stability.

The analysis of the experimental data shows that laminarization of the boundary layer at the airfoil by suction of liquid through a porous casing is possible because of the conditions of equality of the local Reynolds number to its critical value at the point of transition R_t^{**}. In most cases the critical Reynolds number at the point of transition depends on the initial turbulence and airfoil roughness. Both factors are of great practical significance for calculations of a laminar boundary layer of an airfoil with a given roughness moving in a medium with low initial turbulence. In this connexion estimation of possible influence of initial turbulence and surface roughness on the distribution of optimal suction velocity through a special porous surface of airfoils is of great practical interest.

Now we shall determine the local Reynolds number for a boundary layer at the airfoil with suction. For this purpose a set of boundary-layer equations of the zeroth and second moments is used (Kozlov 1962*b*). The zeroth-moment equation is written in the following form:

$$\frac{df}{dx} = \frac{U''}{U'}f + \frac{U'}{U}[a + (B-2)t^{**} - bf], \tag{1}$$

where

$$U' = \frac{dU}{dx}, \quad U'' = \frac{d^2U}{dx^2}.$$

Since

$$f = U'\frac{\delta^{**2}}{\nu} = \frac{\nu U'}{U^2}R^{**2}, \quad \text{where} \quad R^{**} = \frac{U\delta^{**}}{\nu}, \tag{2}$$

the differentiation of equation (2) yields

$$\frac{df}{dx} = \frac{\nu U''}{U^2}R^{**2} - \frac{2\nu U'^2}{U^3}R^{**2} + \frac{\nu U'}{U^2}\frac{dR^{**2}}{dx}. \tag{3}$$

Substitution of formulae (2)–(3) into (1) and some algebraic manipulations give

$$\frac{\nu}{U}\frac{dR^{**2}}{dx} + \frac{\nu U'}{U^2}(b-2)\,R^{**2} - (B-2)\frac{v_0}{U}R^{**} - a = 0. \tag{4}$$

The equation of the second moment is used in the following form (Koslov 1962*b*):

$$\frac{df}{dx} = \frac{U''}{U'}f + \frac{U'}{U}\frac{a}{H_0}(H - H_4t^{**} - H_0cf). \tag{5}$$

Substitution of (2)–(3) into (5) and appropriate algebraic transformations yield

$$\frac{\nu}{U}\frac{dR^{**2}}{dx} + \frac{\nu U'}{U^2}(ac-2)\,R^{**2} + a\frac{H_4}{H_0}\frac{v_0}{U}R^{**} - \frac{aH}{H_0} = 0. \tag{6}$$

Then elimination of v_0/U from (4) and (6) gives the differential equation for the local Reynolds number

$$\frac{dR^{**2}}{dx}+\frac{U'}{U}k_0 R^{**2}-\left[\frac{a}{H_0}\left(\frac{aH_4}{B-2}+H\right)\right]\left[\frac{\nu}{U}\left(1+\frac{a}{B-2}\frac{H_4}{H_0}\right)\right]^{-1}=0, \quad (7)$$

where $$k_0=\left[(ac-2)+\frac{a(b-2)}{B-2}\frac{H_4}{H_0}\right]\left(1+\frac{a}{B-2}\frac{H_4}{H_0}\right)^{-1}.$$

The boundary conditions are

$$R^{**}=R_0^{**} \quad \text{at} \quad x=x_0. \quad (8)$$

Integration of (7) using the boundary conditions (8) gives

$$R^{**2}=\frac{a}{(B-2)H_0+aH_4}\frac{[U(x)]^{-k_0}}{\nu}\int_{x_0}^{x}[aH_4+H(B-2)][U(x)]^{k_0+1}dx + R_0^{**2}\left[\frac{U(x_0)}{U(x)}\right]^{k_0}. \quad (9)$$

Now we determine the critical Reynolds number at the point of transition of a laminar boundary layer into a turbulent one. We adopt Taylor's hypothesis (Taylor 1936) and assume in the subsequent discussion that turbulence in a laminar layer arises because of perturbation with a finite amplitude caused by vortices generated at the airfoil surface because of premature local separations of a laminar layer. In the case considered finite disturbances are introduced into a boundary layer by turbulence in the incoming flow or vortices arising in flows around roughness elements at the airfoil surface.

Premature local separation of a laminar layer is determined approximately by the following relation:

$$\frac{\delta_t^{**2}}{\nu}\frac{1}{\rho U}\left\{\frac{dp}{dx}-\left[\frac{\partial\tilde{p}}{\partial x}\right]\right\}=f_s. \quad (10)$$

The value of the shape factor f_s at the separation point of the boundary layer depends on the intensity of liquid suction through the airfoil surface and is described by the suction parameter. For approximate calculation of the shape factor at the point of separation the following approximate formula (Kozlov 1962*b*) may be used:

$$f_s=f_{s_0}-kt^{**}. \quad (11)$$

Since the pressure fluctuations due to turbulent motions are governed by the law of random effects, the mean square value of external pressure fluctuations is supposed to be governed by the following summation rule:

$$\left[\frac{\partial\tilde{p}}{\partial x}\right]=\left[\frac{\partial\tilde{p}}{\partial x}\right]_1+\left[\frac{\partial\tilde{p}}{\partial x}\right]_2. \quad (12)$$

Turbulence in an incoming flow is assumed to be isotropic. When the point of transition is at a certain distance from the forward stagnation point, then the airfoil surface will not affect the isotropic nature of turbulence at the outer edge of the boundary layer. Then fluctuations of longitudinal pressure gradient are related to the external velocities by the equations of the statistical turbulence theory (Taylor 1936):

$$\left[\frac{\partial \tilde{p}}{\partial x}\right]_1 \sim \frac{\rho u_1'^2}{\lambda_1}, \tag{13}$$

$$\lambda_1 \sim L_\delta \left(\frac{\nu}{u_1' L_\delta}\right)^{\frac{1}{2}}. \tag{14}$$

For a fluctuation flow caused by vortices stalled from the roughness element on the airfoil surface:

$$\left[\frac{\partial \tilde{p}}{\partial x}\right]_2 \sim \frac{\rho u_2'^2}{\lambda^2}, \tag{15}$$

$$u_2' \sim U\left(\frac{d}{\delta_d^{**}}\right), \tag{16}$$

$$\lambda_2 \sim d. \tag{17}$$

Substitution of the critical Reynolds number $R_t^{**} = U\delta_t^{**}/\nu$ and the Taylor parameter $[\epsilon(L/L_0)^{\frac{1}{5}}]$ from relations (12)–(14) into condition (10) reduces this equation to the form

$$R_t^{**} = R_H^{**} + \frac{A_0(f_s+f)^{\frac{1}{2}}}{[\epsilon(L/L_0)^{\frac{1}{5}}]^{\frac{5}{4}}\,[(u_0'/u_1')\,(U_0/U)\,(L_0/L_\delta)^{\frac{1}{5}}\,(\nu/UL_0)^{\frac{1}{5}}]^{\frac{5}{4}} + B_0(Ud/\nu)^{-\frac{1}{2}}\,(d/\delta_d^{**})^2}. \tag{18}$$

The quantity R_H^{**} is introduced into equation (18) to satisfy the condition of damping of perturbations in the boundary layer caused by the external flow turbulence and surface roughness when the critical Reynolds number at the point of transition equals its lower value. This is necessary since as $R_t^{**} \to R_H^{**}$ the fluctuation pressure gradient at the outer edge of the boundary layer does not effect essentially the transition because of rapid damping of fluctuations. Validity of such a transformation is confirmed by experiments.

The following approximate formula (Wieghardt 1954) may be recommended for calculation of lower critical Reynolds number at different pressure gradients and laws of distributed suction:

$$R_H^{**} = \exp(A_1 - B_1 H).$$

To obtain more exact values of the lower critical Reynolds numbers, the study of the stability of the boundary-layer flow is required by the method of small disturbances.

Kozlov (1962*b*) obtained the following approximate formula by 'the three-moment' method:

$$H = H_0 - C_2 t^{**} - mf. \tag{19}$$

The formula is recommended for the following values of parameters:

$$-0{\cdot}08 \leqslant f \leqslant 0{\cdot}08; \quad 0 \leqslant t^{**} \leqslant 0{\cdot}5.$$

The quantities u_1' and L_δ in formula (18) are the root mean square velocity fluctuation and the turbulence scale at the outer edge of the boundary layer, respectively. Since in calculations the turbulence properties of the incoming flow u'_0 and L_0 are usually known, u'_1 and L_δ should be expressed in terms of u'_0 and L_0. To obtain the relations between them, we use the known formulae (Dorodnitsyn & Loitsyansky 1945):

$$\frac{u'_1}{u'_0} \sim \frac{(U/U_0)^3}{1+(U/U_0)^4}, \tag{20}$$

$$\frac{L_\delta}{L_0} \sim \left(\frac{U}{U_0}\right). \tag{21}$$

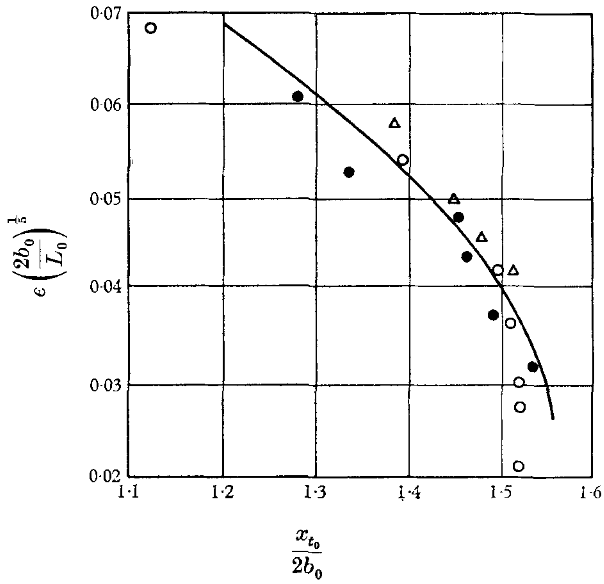

FIGURE 1. Comparison of predicted and experimental data of the Taylor number effect on turbulent transition of laminar boundary layer. —, predicted by formula (22). Schubauer's experiments (see Dryden 1939) for elliptical cylinder: ○, lattice cell 2·5 cm; ●, lattice cell 9·5 cm; △, lattice cell 12·7 cm. b_0, small semi-axis of ellipse.

Substitution of relations (20) and (21) into (18) and necessary manipulations give the following expression for the critical Reynolds number:

$$R_t^{**} = R_H^{**} + \frac{A_0(f_s+f)^{\frac{1}{2}}}{[\epsilon(L/L_0)^{\frac{1}{5}}]^{\frac{5}{4}}\, Re^{-\frac{1}{4}}\Omega(U/U_0) + B_0(Ud/\nu)^{-\frac{1}{2}}\,(d/\delta_d^{**})^2}, \tag{22}$$

where
$$\Omega\left(\frac{U}{U_0}\right) = \frac{(U/U_0)^2}{[1+(U/U_0)^4]^{\frac{5}{4}}}.$$

For a special case of a hydrodynamically smooth surface ($d = 0$), formula (22) agrees with the results of Kozlov (1962*a*).

Equation (22) includes two constant quantities A_0 and B_0 of which attempts at prediction have failed. The analysis of the experimental data has shown that $A_0 = 0{\cdot}22$ and $B_0 = 0{\cdot}21$.

Verification of the semi-theoretical formula (22) has been realized by comparison of the present calculations and Schubauer's experiments (see Dryden 1939) for an elliptical cylinder. The comparison (figure 1) has shown fair agreement of the predicted and experimental data.

In engineering calculations the turbulence scale is not always known since its determination involves precise and tedious measurements. This quantity enters into equation (22) in power $\frac{1}{4}$, so that even a considerable change of its value does not affect the Reynolds number. Besides, the functions Ω and $(Ud/\nu)^{\frac{1}{2}}$ do not change appreciably either.

The comparison of the predicted and experimental data has shown that it may be assumed with an accuracy sufficient for practical calculations that

$$A_0\left(\frac{U_0 L}{\nu}\right)^{\frac{1}{4}}\Omega^{-1} \simeq 2\cdot 0, \tag{23}$$

$$\frac{A_0}{B_0}\left(\frac{Ud}{\nu}\right)^{\frac{1}{2}} \cong 2250. \tag{24}$$

This considerably simplifies formula (18), which will be in a very convenient form for practical application:

$$R_t^{**} = R_H^{**} + \frac{C_1(f_s+f)^{\frac{1}{2}}}{\epsilon^{\frac{5}{4}} + D_1(d/\delta_d^{**})^2}. \tag{25}$$

The constants C_1 and D_1 in (25) are $1\cdot 88$ and $0\cdot 25\times 10^{-3}$, respectively.

The predicted and experimental values of critical Reynolds numbers at the point of transition are plotted in figure 2 versus the amount of disturbance (see (25)). Accounting for the scatter of the experimental points, we may consider the agreement to be quite satisfactory except that portion of the curve which corresponds to inconsiderable (below $0\cdot 1$ %) turbulence of the incoming flow.

In figure 3 the predicted and experimental data of critical Reynolds numbers for plates with various roughnesses of the surface are presented. The experiments (Tani, Hama & Mituishi 1954; Tani, Juchi & Yamamoto 1954; Feindt 1956) were carried out with small ($\epsilon \cong 0\cdot 15$ %) and high ($\epsilon \simeq 0\cdot 8$ %) turbulence of the incoming flow. The comparison shows satisfactory agreement of the values predicted by formula (25) with the experimental data except in the case of very small turbulence and roughness ($\epsilon \leqslant 0\cdot 15$ % and $d/\delta_d^{**} = 0\cdot 75$). In this situation the discrepancy may be attributed to the fact that the procedure adopted is not valid for transition of a laminar boundary layer into a turbulent one for very small velocity disturbances caused by initial flow turbulence or surface roughness.

It is of interest to note that comparison of the present data with the plots reported by Crabtree (1958), Preston (1958) and Schlichting (1959) for prediction of the points of transition shows fair agreement with the results predicted by (25) for $\epsilon = 0\cdot 15$–$0\cdot 35$ %. This fact is another proof of the validity of (25) since the data used for these plots are obtained within the range of turbulence of the incoming flow considered.

Substitution of expressions (11), (18) and (19) into formula (25) and simple algebraic transformations yield the final expression for the critical Reynolds number at the point of transition:

$$R_t^{**} = \exp(\mathbf{A}_1 - \mathbf{B}_1 H) + \frac{c_1[f_{s_0} + H/m(c_2/m-k)\,t^{**} - H/m]^{\frac{1}{2}}}{\epsilon^{\frac{5}{4}} + \mathbf{D}_1(d/\delta_d^{**})^2}. \tag{26}$$

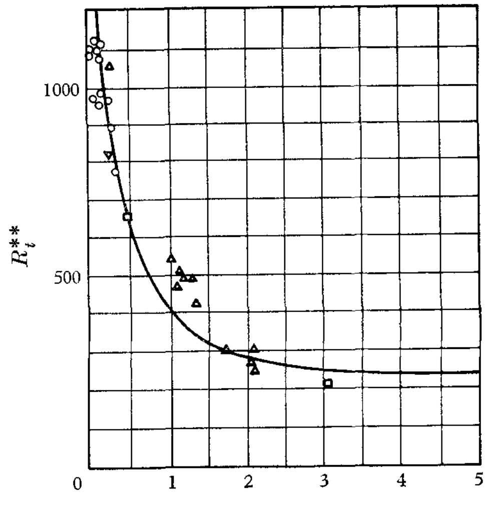

FIGURE 2. Comparison of predicted and experimental data of initial turbulence effect on the transition-point critical Reynolds number: —, predicted by formula (25); □, experiments (Dryden 1936); △, experiments (Hall & Hislop 1938); ▽, experiments (Wright & Bailey 1939); ○, experiments (Schubauer & Skramstad 1947).

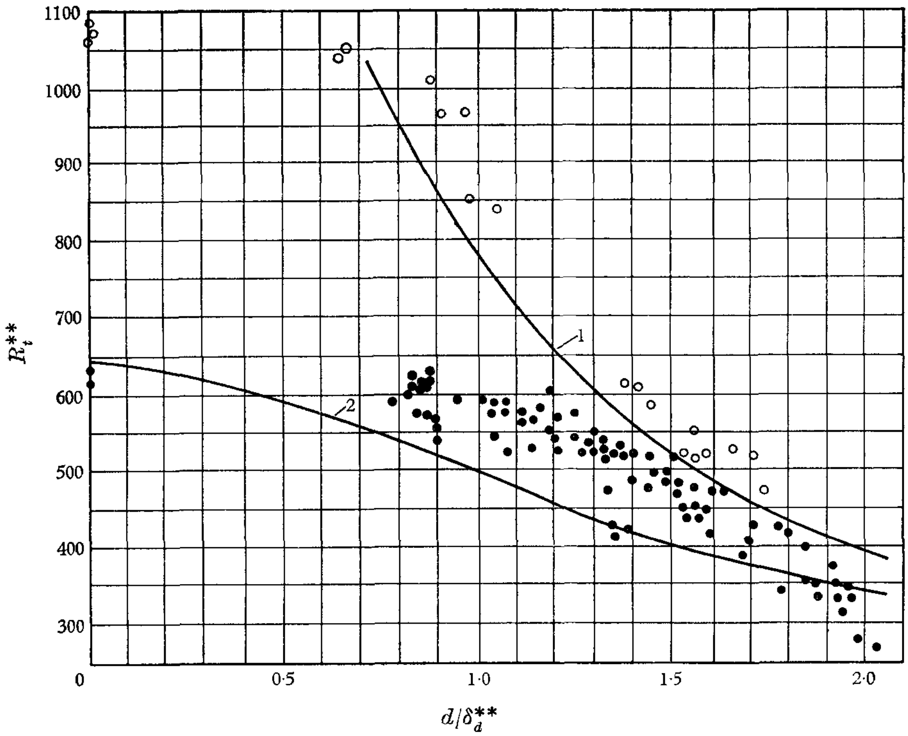

FIGURE 3. Comparison of predicted and experimental data on combined effect of initial turbulence and surface roughness on transition-point critical Reynolds number: —, predicted by formula (25); ○, experiments (Tani, Juchi & Yamamoto 1954); ●, experiments (Feindt 1956); 1, and 2, predicted values for the conditions of these experiments, respectively.

Further, with the critical Reynolds number at the point of transition known, the zeroth-moment equation (4) gives the formula for calculation of the optimal distribution of the suction velocity:

$$\frac{v_0}{U} = \frac{\nu}{U}\frac{1}{B-2}\frac{1}{R^{**}}\frac{dR^{**2}}{dx} + \frac{\nu U'}{U^2}\frac{b-2}{B-2}R^{**} - \frac{a}{B-2}\frac{1}{R^{**}}. \tag{27}$$

The first term in (27) will be obtained from differential equation (7). After simple manipulations we have

$$\frac{v_0}{U}\frac{1}{B-2}\frac{1}{R^{**}}\frac{dR^{**2}}{dx} = -\frac{\nu U'}{U^2}\frac{k_0}{B-2}R^{**} + \frac{a}{B-2}\frac{aH_4+H(B-2)}{H_0(B-2)+aH_4}\frac{1}{R^{**}}. \tag{28}$$

Substitution of expression (28) into (27) and the necessary calculations give the finite form of the formula for optimal distribution of the suction velocity through the porous surface along the airfoil chord

$$\frac{v_0}{U} = \frac{\nu U'}{U^2}\frac{b-2-k_0}{B-2}R^{**} + \frac{a(H-H_0)}{H_0(B-2)+aH_4}\frac{1}{R^{**}}. \tag{29}$$

Having found the optimal distribution of the suction velocity from formula (29) by any available method (for example, that proposed by Kozlov 1962*b*), we may calculate the remaining properties of the boundary layer and the profile resistance of the airfoil.

To satisfy the condition of equality of the local Reynolds number to its critical value at the point of transition, the method of successive approximations is recommended for calculation. In the first approximation the longitudinal external velocity gradient should be neglected, and the optimal distribution of the suction velocity along the chord of the airfoil is to be found by formula (29). In the second approximation the values of R_t^{**} and d/δ_d^{**} are specified appropriately to the suction law found in the first approximation. Repeated application of successive approximations allows calculation of the optimal distribution of the suction velocity as well as the remaining boundary-layer properties and the profile resistance of the airfoil. Practical calculations have shown that, for airfoils with the chord to thickness ratio above seven at angles of incidence close to the optimal values, the second and the third approximations are practically the same.

Consider in detail the particular case of a plate, for which the fundamental predicted equations (9), (26) and (29) are simplified considerably and upon simple transformations become of the form

$$R^{**2} = \left[2\left(\zeta + \frac{d+1}{c_2}H_0\right) - \frac{2(d+1)}{c_2}H\right](R_x - R_{x_0}) + R_0^{**2}, \tag{30}$$

$$R_t^{**} = \exp(A_1 - B_1 H) + \frac{c_1}{\epsilon^{\frac{5}{4}} + D_1(d/\delta_d^{**})^2}\left[f_{s_0} - \frac{k}{c_2}(H-H_0)\right]^{\frac{1}{2}}, \tag{31}$$

$$\frac{v_0}{U} = \frac{\nu}{U}\frac{1}{B-2}\frac{1}{R^{**}}\frac{dR^{**2}}{dx} - \frac{a}{B-2}\frac{1}{R^{**}}. \tag{32}$$

The formulae (30), (31) and (32) allow the necessary calculations to be carried out. In figures 4 and 5 the optimal distribution of the suction velocity is plotted versus the Reynolds number R_x for various values of the initial turbulence of the incoming flow and surface roughness.

The analysis of the data presented in these figures shows that optimal distribution of the suction velocity and the total suction flow rate depend essentially on the initial turbulence and the surface roughness. With small initial flow turbulence ($\epsilon = 0{\cdot}2\,\%$), the surface roughness has a considerable effect on the optimal suction. The required quantity of liquid to be sucked increases with roughness. With large initial turbulence ($\epsilon = 2\,\%$) change of the surface roughness has no essential effect on the optimal distribution of the suction velocity.

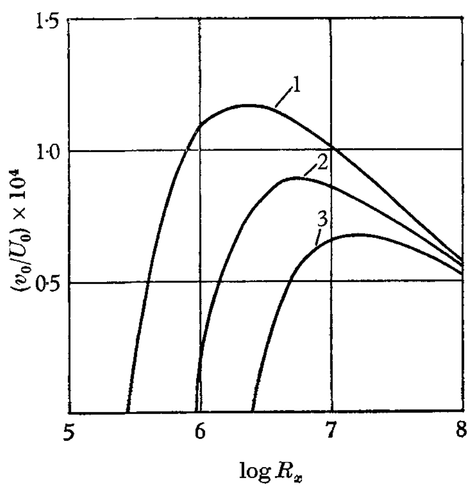

FIGURE 4

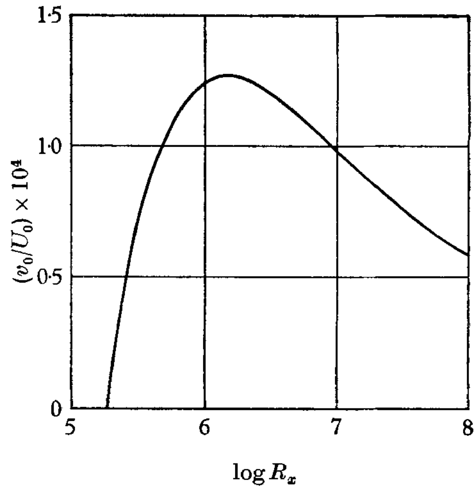

FIGURE 5

FIGURE 4. Optimal distribution of suction velocity for a plate versus the Reynolds number with initial turbulence $\epsilon = 0{\cdot}2\,\%$: 1, $d/\delta_d^{**} = 4$; 2, $d/\delta_d^{**} = 2$; 3, $d/\delta_d^{**} = 1$.

FIGURE 5. Optimal distribution of suction velocity for a plate with initial turbulence $\epsilon = 2\,\%$ and surface roughness $d/\delta_d^{**} = 1 \div 4$.

The drag coefficient of laminar friction with the optimal distribution of the suction velocity is calculated for a boundary layer on a plate as

$$\zeta_f = \frac{2\int_0^x \tau_w\,dx}{\rho U_0^2 x} = \frac{2}{R_x}\int_0^{R_x} \frac{\zeta v_0/U_0}{t^{**}}\,dR_x. \tag{33}$$

Kozlov (1962*b*) showed that in the case of suction from the boundary layer

$$\zeta = \zeta_0 - dt^{**}. \tag{34}$$

Using formula (34), we transform expression (33) into the following form:

$$\zeta_f = \frac{2\zeta_0}{R_x}\int_0^{R_x} \frac{dR_x}{R^{**}} - \frac{2d_0}{R_x}\int_0^{R_x} \frac{v_0}{U_0}\,dR_x. \tag{35}$$

Bearing in mind that for $Re_x \leqslant R_{x_0}: v_0/U_0 = 0$ and $R^{**} = 0{\cdot}664\sqrt{R_x}$, we obtain finally

$$\zeta_f = \frac{1{\cdot}328\sqrt{R_{x_0}}}{R_x} + \frac{0{\cdot}44}{R_x}\int_{R_{x_0}}^{R_x}\frac{dR_x}{R^{**}} - \frac{1{\cdot}12}{R_x}\int_{R_{x_0}}^{R_x}\frac{v_0}{U}dR_x. \tag{36}$$

Both integrals in formula (36) have been calculated by graphical integration using the data of figures 4 and 5.

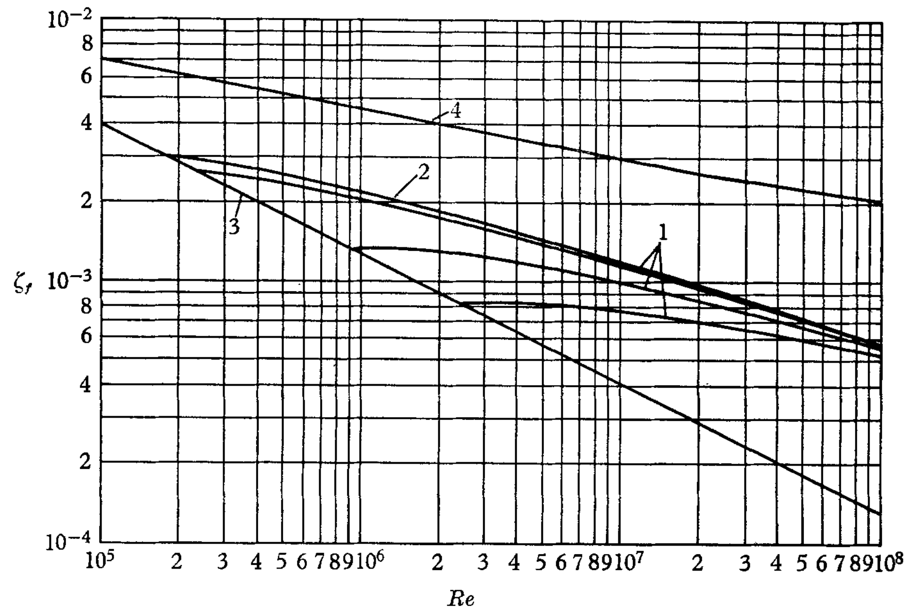

FIGURE 6. Friction coefficient of a plate versus the Reynolds number; 1 and 2, optimal suction for $\epsilon = 0{\cdot}2\,\%$ and $\epsilon = 2\,\%$, respectively; 3, Blasius's data for a laminar flow; 4, Prandtl–Schlichting's data for a turbulent flow.

In figure 6 numerical values of the friction drag coefficients for a plate with optimal suction of a laminar boundary layer are plotted versus the Reynolds number R_x. In this diagram, figure 1 is used for designation of the friction coefficient of a laminar friction of plates with the optimal suction velocities for small initial turbulence ($\epsilon = 0{\cdot}2\,\%$), and figure 2 for high initial turbulence ($\epsilon = 2\,\%$). In this figure the coefficients of laminar and turbulent friction for plates without suction of the boundary layer according to Blasius's (curve 3) and Prandtl–Schlichting's data (curve 4) are also presented.

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